



PAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAM	LEVEL: 7
COURSE CODE: MMO701S	COURSE NAME: MATHEMATICAL MODELLING 1
SESSION: JUNE 2019	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF. S. A. REJU
MODERATOR:	PROF. O. D. MAKINDE

INSTRUCTIONS
<ol style="list-style-type: none">1. Attempt ALL the questions.2. All written work must be done in blue or black ink and sketches must be done in pencils.3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [20 MARKS]

- (a) Discuss the method of Conjecture in Mathematical modelling and hence by employing the Conjecture method, show that the solution of the dynamical system

$$a_{n+1} = ra_n + b, \quad r \neq 1 \quad (1.1)$$

Is given by

$$a_k = r^k c + \frac{b}{1-r} \quad (1.2)$$

for some c (which depends on the initial condition). [6]

- (b) Given the following experimental data from a spring-mass system:

Mass	50	100	150	200	250	300	350
Elongation	1.20	1.65	2.00	3.15	4.200	5.21	6.00

Formulate two different models that estimate the proportionality of the elongation to the mass, clearly showing how your proportionality constant is obtained for each model (correct to 4 decimal places). [14]

QUESTION 2 [25 MARKS]

- (a) A certain drug is effective in treating a disease if the concentration remains above 100 mg/L. The initial concentration is 640 mg/L. It is known from laboratory experiments that the drug decays at the rate of 20% of the amount present each hour.
- (i) Formulate a model representing the concentration at each hour.
- (ii) Build a table of values and determine when the concentration reaches 100 mg/L.

[8.5]

- (b) Consider the following table showing the experimental data of the growth of a micro organism

n	0	1	2	3	4	5	6
y_n	10.6	18.3	29.2	45.5	71.1	120.1	174.6
Δy_n	8.7	11.7	16.3	23.9	52	55.5	85.6

where n is the time in days and y_n is the observed organism biomass.

- (i) Formulate a linear model for the above organism and show that the model predicts an increasing population without limit.
- (ii) Assume that contrary to your model prediction in (i), there is a maximum population of 320. Hence formulate a nonlinear dynamical system model for the organism using your constant obtained from an appropriate ratio similar to the example given in class, for $n = 3$ in the above data. [16.5]

QUESTION 3 [30 MARKS]

(a) Consider the motion of a raindrop from a motionless cloud, subject to force F_d due to the air resistance and the gravitational force F_g . Employing appropriate geometric similarity relationships, state at least four essential assumptions and any physical law to determine the relationship between the terminal velocity of the raindrop and its mass.

[10]

(b) Consider the following table of data:

X	1	2.3	3.5	4.5	6.5	7.0
y	3.5	3.2	5.5	6.2	4.5	7.5

(i) Estimate the coefficients of the straight line $y = ax + b$ such that the sum of the squared deviations of the data points and the line is minimised.

(ii) If the largest absolute deviations for the Chebyshev's criterion and that of the Least Squares criterion are given respectively by c_{max} and d_{max} , define them and then compute their values including their least bound D to express their relationship for the above data and the model line.

[20]

QUESTION 4 [25 MARKS]

(a) A sewage treatment plant processes raw sewage to produce usable fertilizer and clean water by removing all other contaminants. The process is such that each other 15% of remaining contaminants in a processing tank are removed.

- i. What percentage of the sewage would remain after half a day?
- ii. How long would it take to lower the amount of sewage by half?
- iii. How long until the level of sewage is down to 12% of the original level?

[12]

(b) Consider an annuity where a savings account pays a monthly interest of 1% on the amount present and the investor is allowed to withdraw a fixed amount of N\$1000 monthly until the account is depleted. What is the solution of the dynamical system model for the annuity problem and how much of the initial investment will be needed to deplete the annuity in 20 years?

[13]

END OF QUESTION PAPER

TOTAL MARKS = 100